PAP-003-001513

PAP-003-001513 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October / November - 2018

Mathematics : Paper-BSMT-501(A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003
Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions in short : **20**

(1) Define metric space.

(2) Give an example of a subset of metric space R which is not open and closed.

(3) Define limit point.

(4) Define perfect (complete) set.

(5) If E=[1,2] is a subset of metric space R then find E^0.

(6) Define Lower Riemann Integration.

(7) If $f(x)=\frac{20}{x}$, $x\in[2,20]$ and partition $P=\{1,4,5,20\}$ then find $\|P\|$.

(8) State fundamental theorem of Riemann integration.

(9) State Darboux's theorem.

(10) State general form of first mean value theorem.

(11) Define Permutation.

(12) Define Alternating Group.

(13) Define Right coset.

(14) If $f=(4,5)(2,3,7)$ then find $O(f)$, where $f \in S_8$.

(15) For group $(Z_5,+_5)$, $O(4)=$......

(16) Define Inner automorphism.

(17) For the group $(Z_6,+_6)$, find generators of Z_6.

(18) Define Index of subgroup H in group G.

(19) Define Factor group.

(20) Define Isomorphism of groups.

2 (a) Answer any three : **6**

(1) Define norm of partition and finer partition.

(2) If (X,d) is a metric space and $A,B \subset X$ and $A \subset B$ then $Int\, A \subset Int\, B$.

(3) Obtain border set of the subset $(-1,1)$ of metric space R.

(4) Determine whether set $\{x \in R / x^2 - 2x - 1 = 0\}$ is open or closed set.

(5) If $f(x)=\dfrac{20}{x}$, $x \in [2,20]$ and partition $P=\{2,4,5,20\}$ then find $U(P,f)$.

(6) Evaluate : $\lim_{n\to\infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$.

(b) Answer any three : **9**

(1) If f is increasing function on $[a,b]$ then prove that f is R-integrable.

(2) If (X,d) is a metrix space and $A,B \subset X$ the prove $(A \cap B)' = A' \cap B'$.

(3) Prove that every continuous function f on $[a,b]$ is Riemann integrable on $[a,b]$.

(4) If f and g are R-integrable on $[a,b]$ then prove that f+g is also R-integrable on $[a,b]$.

(5) Prove that every finite subset of any metric space is a closed set.

(6) State and prove principle of Housdorff's in metric space.

(c) Answer any two : **10**

(1) In usual notation prove that $\overline{E}$ is a closed set in metric space.

(2) State and prove necessary and sufficient condition for a bounded function to be R-integrable.

(3) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x}\, dx \leq \frac{\pi^3}{6}$.

(4) Prove that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.

(5) Prove that $\frac{3}{4}$ is in cantor set.

3 (a) Answer any three : **6**

(1) If $\sigma = (1\,3\,4)(2\,5), \sigma \in S_5$ then find σ^{-1}.

(2) If $f : R \to R$ is defined as $f(x) = x^2$ then check whether f is a permutation or not.

(3) Prove that intersection of two subgroups of a group is again a group.

(4) Prove that the identity element is only idempotent element in any group.

(5) If $a^2 = e$ for each element a of a group G then show that G is commutative.

(6) Prove that every element of a finite group is of finite order.

(b) Answer any three : **9**

(1) Let $H \le G$ and let $a, b \in G$ then show that $aH = Ha \Leftrightarrow ab^{-1} \in H \cdot$

(2) Prove that a group of prime order is cyclic.

(3) If H is a normal subgroup of group G with $i_G(H) = m$ then prove that $a^m \in H; \forall a \in G \cdot$

(4) Draw the lattice diagram of Z_8.

(5) If H and K are any two subgroups of group G such that (O(H,O(K)) = 1 then show $H \cap K = \{e\}$.

(6) If G is a finite group, then $O(a)/O(G); \forall a \in G$.

(c) Answer any two : **10**

(1) State and prove Cayley's theorem.

(2) Prove that any two right cosets of a subgroup H in group G are either identical or disjoint.

(3) State and prove Lagrange's theorem for finite groups.

(4) Prove that every permutation $f \in S_n$ can be expressed as a composition of two disjoint cycles.

(5) Show that $(R, +) \cong (R_+, \bullet)$.